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Air Levitation State Feedback Control Using Pole Placement and Kalman Filter ⋆

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Abstract: State feedback controller design and implementation of Kalman filter for Air levitation system is studied in simulation environment (Matlab/Simulink) and on real laboratory plant, low cost Floatshield device developed recently at our University and having an open source support, Takács (2022). Nonlinear model and its linearized state space representation are used for a pole placement controller design. The main focus is on Kalman filter implementation via its equations, which can help better understanding of its principle. Simulation results received for linear and nonlinear systems are compared with real plant experiments. The authors believe that the presented control design topics and corresponding tasks can contribute to developing students' skills in control design and further enhance educational potential of hands-on laboratories using low cost air levitation device.

Keywords: air levitation system, control education, state-feedback control, Kalman filter, pole-placement control.

1. INTRODUCTION

Air or magnetic levitation belongs to interesting physical phenomena with considerable application potential, for example in the transport of materials or the positioning of various objects. Due to its nonlinear and inherently unstable dynamics, levitation systems represent a challenging education platform in automatic control courses. Air levitation plants are popular in control laboratories as low-cost devices suitable for considering various control aspects and approaches, see Chacon et al. (2017), Takács et al. (2020), Espí et al. (2017) and Chołodowicz and Orłowski (2017) to name a few. Various control design approaches have been used to control a position of the levitating ball, most frequently PID control and its variants, as in Chacon et al. (2017), Takács et al. (2020), Chacón et al. (2018), Pham et al. (2022), Bomfim et al. (2021), Chołodowicz and Orłowski (2017). Adaptive MRAC modification to PID controller is introduced in Bomfim et al. (2021), in Pham et al. (2022), PID controller is implemented using PLC. In Chaos et al. (2020), the authors propose robust controller inspired by a sliding mode approach.

Position control systems often use state feedback as an efficient tool to consider both stability and performance requirements. State feedback was successfully used for positioning in magnetic levitation system, Bojan-Dragos et al. (2016), Hypiusova and Rosinova (2021). This motivated us to use an air levitation system for teaching a state feedback control. In most real systems, not all state variables are measurable, then, observers are included in control system. In Rosinová and Schwarz (2023) we presented state feedback control and its implementation with a deterministic state observer on a low cost laboratory Floatshield system developed at Slovak University of Technology, as an educational device with open source documentation, see Takács et al. (2020) and Takács (2022).

In this paper we further develop education potential of the air levitation system and propose its use to teach advanced state feedback controller design employing Kalman filter to estimate system states. Our focus is on implementing the Kalman filter using its equations, not just taking some off-the-shelf Kalman filter as a "black box" for a state estimation. The aim is to achieve basic understanding how the Kalman filter works, how to implement it and support this knowledge by practical experiments on the air levitation system. The paper is organized as follows. First, air levitation laboratory plant is briefly introduced and its nonlinear and linearized models are recalled. The main part is devoted to a state feedback controller design using a discrete-time robust pole placement approach. Then, a simulation model of Kalman filter equations in Simulink is presented, which clarifies its functioning. State feedback and Kalman filter designs are tested on simulation model and then on a laboratory plant. The modelling, control and state estimation sections are complemented by tasks, proposed to achieve the relevant learning objective.

2. PRELIMINARIES

Learning objectives considered in this paper assumes that students have basic control background. Namely, state-space representation of dynamic systems, linear state feedback control (LQ and pole placement), linearization based on Jacobian and identification of nonlinear systems around a working point. In our recent paper Rosinová and Schwarz

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(2023), details of identification procedure of considered laboratory air levitation plant were presented and a state feedback pole placement controller was designed with basic deterministic observer. In this work, we focus on Kalman filter design and implementation, modelling topic is only briefly recalled. To distinct air levitation simulation model from a laboratory plant, below we denote the former as ALS and the latter as Floatshield.

3. AIR LEVITATION SYSTEM

The air levitation system is a vertical tube with a levitating object (ball) moving inside it; laboratory plant Floatshield, Takács et al. (2020) is in Figure 1. System

Fig. 1. Floatshield laboratory plant, Takács et al. (2020).

input is an air flow blown by a fan from below and the output is the position (height) of the levitating ball in the tube. The control aim is to position the ball in the tube. Modelling this system is not an easy task. We consider a simplified model based on first principles, Chacon et al. (2017), Takács et al. (2020). Dynamics of the ALS based on second Newton law is described by equation (1)

$$m\frac{d^2y}{dt^2} = \frac{1}{2}\rho A C_d (v_a - \frac{dy}{dt})^2 - mg. \tag{1}$$

where m, ρ, A, C_d, g are constants: m is a ball mass, ρ is the density of the air in the tube, A is the area of the ball exposed to the air flow, C_d is a drag coefficient, g is the gravitational constant; $y(t)$ is an output variable - vertical position of the ball in the tube (height) and $v_a(t)$ is the air velocity in the tube.

The air flow is delivered by a fan, its dynamics is

$$\frac{dv_a}{dt} = -\frac{1}{\tau_1} v_a + \frac{K_f}{\tau_1} u. \tag{2}$$

where $u(t)$ is input power sent to the fan, τ_1 is a time constant of the fan and K_f its gain.

Equations (1) and (2) describe the overall dynamics of the ALS system. Since the state feedback control will be studied, (1) is rewritten to two state equations, where the term for drag force is reformulated to include also the sign corresponding to the direction of the ball movement. The nonlinear ALS model is summarized as

$$\frac{dx_1}{dt} = x_2$$

$$\frac{dx_2}{dt} = \frac{\alpha}{m}(v_a - \frac{dy}{dt})|v_a - \frac{dy}{dt}| - g \tag{3}$$

$$\frac{dx_3}{dt} = -\frac{1}{\tau_1} x_3 + \frac{K_f}{\tau_1} u. \tag{4}$$

where $x_1(t) = y(t), x_3(t) = v_a(t)$, $| * |$ represents absolute value of $*$.
Presented state model can be linearized using standard approach based on Jacobian, resulting in

$$\frac{dx}{dt} = Ax(t) + Bu(t), \quad y(t) = Cx(t) \tag{5}$$

where state vector $x(t) = [x_1(t), x_2(t), x_3(t)]^T$ and

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{1}{\tau_2} & \frac{1}{\tau_2} \\ 0 & 0 & -\frac{1}{\tau_2} \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ \frac{K_f}{\tau_1} \end{bmatrix} \tag{6}$$

Output matrix $C = [1, 0, 0]$.

Tasks within modelling part can be formulated as follows.

- Identify a linearized ALS model. Since ALS is unstable system, identification is performed in a closed loop with stabilizing (default) controller. Model parameters τ_1, τ_2, K_f can be found by numerical optimization to fit the measured step response. More details can be found in Rosinová and Schwarz (2023).
- Build Simulink models both for the linear model (5) and the nonlinear one (4). Compare the responses of Simulink models with Floatshield ones. (Closed-loop responses with default stabilizing controller should be compared.)

4. STATE FEEDBACK CONTROL

There exist various state feedback controller design approaches; linear quadratic (LQ) and pole placement belong to basic ones taught in standard control courses. In ALS state feedback controller design, three main aspects should be considered.

- The implementation for a real system requires a discrete-time model, therefore it seems more appropriate to design a controller in a discrete-time domain.
- Positioning the levitating ball requires to include an integration term in the controller. Therefore, an augmented discrete-time state space model including te integration term is used in the pole-placement control.
- It is difficult to prescribe a precise closed-loop pole location, therefore a pole region approach considered in robust control provides useful tool to specify the required performance.

These aspects are studied in the following two subsections.

4.1 Discrete-time state space model for state feedback control with integration term

Consider the linear discrete-time system recalculated from (5) for appropriate sampling time (for Floatshield, sampling time $Ts = 0.025s$ is used)

$$x(k+1) = A_d x(k) + B_d u(k), \quad y(k) = Cx(k) \tag{7}$$

where $x(k), u(k), y(k)$ are discrete-time (sampled) counterparts to continuous-time state model variables; matrices A_d, B_d are recalculated from A, B from (6) for sampling time Ts. The proposed control structure is in Figure 2 and consists of a state feedback, an integration term in forward path and the Kalman filter to estimate states for the state feedback. When the state feedback control is combined with an integration term for control error in the forward path, then the corresponding controller is designed for the augmented system, where a state corresponding to discrete-time integration (summation) is added. The augmented discrete-time model is then in the form

$$x_a(k+1) = A_{da} x_a(k) + B_{da} du(k), \quad y_a(k) = C_a x_a(k) \tag{8}$$

where the augmented state vector $x_a(k) = [x(k), x_i(k)]^T$, $y_a(k) = [y(k), y_i(k)]^T$, $x_i(k) = y(k) - w$ and $y_i(k)$ corresponds to a discrete-time integration term, w is the reference for ball position. Augmented matrices are

$$A_{da} = \begin{bmatrix} A_d & 0 \\ -C & 1 \end{bmatrix}, \quad B_{da} = \begin{bmatrix} B_d \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} C & 0 \\ 0 & 1 \end{bmatrix}, \tag{9}$$

The control problem is to find a gain matrix $K_a = [K_p, K_i]$ for the state feedback control law

$$u(k) = -K_a x_a(k) \tag{10}$$

so that the closed-loop augmented system is stable with a required performance.

4.2 Robust discrete-time pole placement control

Pole placement belongs to efficient control design strategies, since poles determine the overall system dynamics and performance. Performance indices as overshoot, settling time, relative damping or decay rate depend on pole position in the complex plane, basically, on a stability degree and a relative damping. Pole placement controller design can be done in two ways. The most simple is to guess the appropriate closed loop pole location - it is recommended to place the poles close to stability border and to real axis so that the control variable is not too big and response is well damped. However, to prescribe the exact pole position is often difficult and in a real plant control it is more advantageous to prescribe an appropriate region where the closed-loop system poles should be placed. The alternative based on pole regions approach is more systematic and general but a bit more complicated. Below, the principle of the latter is briefly summarized. In the proposed control course we avoid the theory behind regional pole placement and use the results that we have developed recently for a discrete- time state feedback pole regions, Hypiusova and Rosinova (2021). Students can concentrate on the essence of pole region approach - choice of an appropriate pole region, and use already existing computational tools to calculate state feedback gain matrix for the prescribed stability degree and relative damping. It is important to note substantial differences between continuous-time and discrete-time systems. While the pole region corresponding to the stability degree is in both cases simple and convex: shifted left half-plane for continuous-time and circle centred in $[0, 0]$ with radius $r < 1$ for discrete-time cases, situation for the relative damping ϕ is more complicated. It is important to note that the relative damping turned out to be important factor in ALS control, therefore it has to be included in the performance requirements. The discrete-time counterpart to the convex cone pole region is a non-convex region bounded by logarithmic spirals, see Figure 3. To simplify the pole-placement design (so that LMI based computation can be used), the nonconvex region (bounded by the blue line) is approximated by the convex inner approximation (bounded by the black line). Input parameters for a state feedback gain computation are a prescribed relative damping ϕ and x_e representing the design parameter, see see Figure 3. For ALS, relative damping 30° is recommended and stability degree corresponding to r about 0.98-0.99. It is important to note that the pole placement controller is designed for the augmented system (8) and the resulting gain matrix $K_a = [K_p, K_i]$ is for implementation divided into its feedback part K_p and integrator gain K_i.

Tasks within the state feedback control part can be formulated as follows.

- Recalculate the discrete-time state space model for appropriate sampling time. Set-up the augmented state model including integration term for a controller.
- Choose appropriate relative damping (about 30°) and corresponding poles (pole region) for a discrete-time closed loop. Design a pole placement state feedback controller for the augmented system.
- Test and tune the controller for the nonlinear system. Use simulation model in Figure 2; for a state feedback use the nonlinear model states.

5. KALMAN FILTER AND ITS SIMULINK IMPLEMENTATION

The central point and main learning objective of the proposed control design topic is to enhance the understanding of the fundamentals of the discrete-time Kalman filter and to gain basic skills in its software implementation.

Real systems are always influenced by uncertainties and noise. The Kalman filter is based on probabilistic models and considers noisy measurement when estimating the system state. System model with additional noise is considered as follows

$$x(k+1) = A_d x(k) + B_d u(k) + Gn(k), \quad y(k) = Cx(k) + v(k), \tag{11}$$

where $n(k)$ is a white noise influencing the internal states with covariance $Q > 0$, $v(k)$ is a measurement white noise with covariance $R > 0$,

The Kalman filter belongs to important knowledge of control designer and is studied in many books on control theory, see e.g. Lewis et al. (2012); a nice Kalman filter tutorial is in Becker (2019). Basically, state estimate in Kalman filter is computed in two phases: prediction (apriori estimates) according to a model and correction

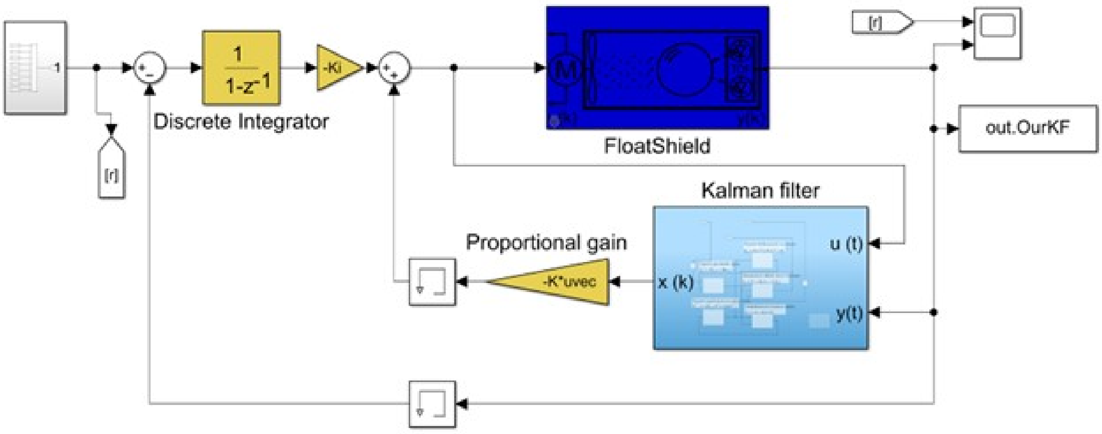

Fig. 2. Closed loop system for a state feedback PI control with Kalman filter. Dark blue: air levitation system, light blue: Kalman filter, yellow: state feedback discrete-time PI controller.

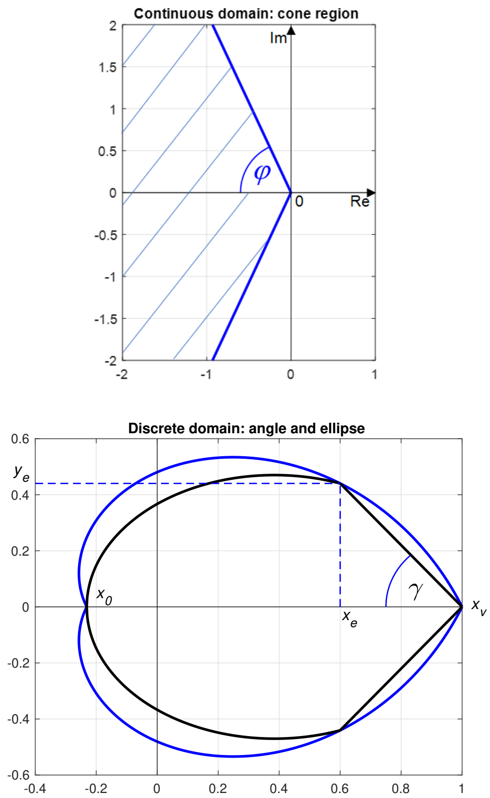

Fig. 3. Pole domains for a prescribed relative damping ϕ for a continuous-time (upper figure) and a discrete-time (lower figure) cases, Hypiusova and Rosinova (2021).

(posterior estimates) based on measured values and an error covariance matrix.

It is important to appropriately denote all used symbols not to loose orientation in variables and their predicted (apriori) and posterior estimates. Used notation is summarized in Table 1,

In the apriori phase, state and error covariance predictions are computed. The state prediction is calculated from the system model, in our case from the linearized one

Table 1. Notation

variable	notation
predicted state estimate	$\widehat{x}^-$
posterior state estimate (update)	$\widehat{x}$
predicted covariance estimate	P^-
covariance update	P
Kalman gain	K

$$\widehat{x}^-(k) = A_d\widehat{x}^-(k-1) + B_d u(k-1). \tag{12}$$

The predicted error covariance matrix estimate is computed recursively, starting from $P_0 = I$, where I denotes identity matrix, as

$$P^-(k) = A_d P(k-1) A_d^T + GQG^T \tag{13}$$

In the posterior phase, Kalman gain $K(k)$ is calculated

$$K(k) = P^-(k)C^T(CP^-(k)C^T + R)^{-1} \tag{14}$$

Having Kalman gain, state can be updated considering the actual output measurement according to

$$\widehat{x}(k) = \widehat{x}^-(k) + K(k)(y(k) - C\widehat{x}^-(k)) \tag{15}$$

where $y(k)$ represents the actual output measurement.

Finally, the error covariance matrix is updated

$$P^(k) = (I - K(k)C)P^-(k) \tag{16}$$

The Kalman filter can be modelled in Simulink using equations (12)-(16) as shown in Figure 4. After modelling Kalman filter, all parts of the overall closed loop model Figure 2 are finished.

Tasks for Kalman filter modelling can be formulated as follows.

- Build the Kalman filter model according to equations (12)-(16). The overall structure can be seen in Simulink model in Figure 4.
- Include the Kalman filter model into the closed-loop according to Figure 2 with the state feedback controller designed in previous Section, and test its

functioning. Compare the responses of the closed-loop with the direct state feedback and with the Kalman filter.

- Add the additional noise to the model, according to (11). Modify the noise covariance matrices Q, R and study their influence on the corresponding responses.

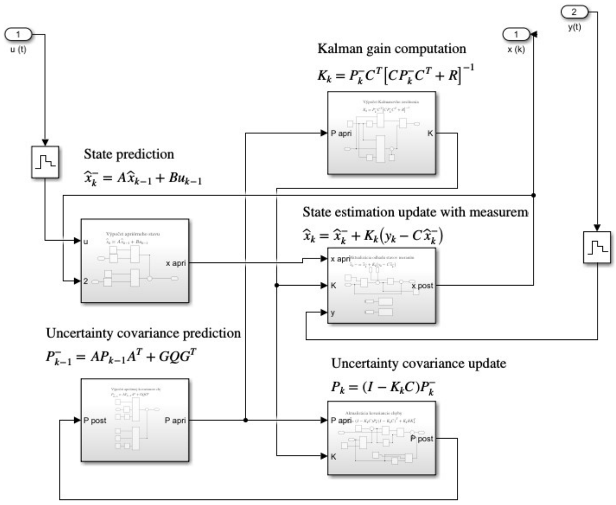

Fig. 4. Simulink model of the Kalman filter.

6. SIMULATION RESULTS

This section is devoted to illustration of simulation and experimental results of the tasks presented in previous sections. We use plant parameters of the Floatshield from Rosinová and Schwarz (2023): $K_f = 11, \tau_1 = \tau_2 = 0.3673$ identified before. In the modelling task, we recalculated corresponding continuous state space model (5) to a discrete-time form (7) using sampling period $T_s = 0.025s$. For the pole placement controller design we used augmented model (8). The required closed loop poles were set to $PP = [0.9208 + 0.0023i, 0.9208 - 0.0023i, 0.8936 + 0.0022i, 0.8936 - 0.0022i]$. The resulting feedback gain matrix was received using standard Matlab function (place) as $K_{aug} = [2.5682, 0.6115, 0.3097, 0.0597]$. This feedback gain matrix was split into the proportional feedback gain $K = [2.5682, 0.6115, 0.3097]$ and the integral term $K_i = 0.0597$. Kalman filter was considered with $G = I$ and noise covariance matrices $Q = diag(5, 1000, 1000), R = 25$. The designed state feedback pole placement controller and Kalman filter were tested by simulations on the linear and nonlinear models to note the differences between them.
First, closed-loop responses for the pole placement state feedback controller are compared for the linear and nonlinear models with the full state accessible and the nonlinear model with states estimated by the Kalman filter. In model simulations, results for the full state are very close to the results obtained using Kalman filter to estimate the states, see Figure 5. We compared our Simulink implementation of the Kalman filter, Figure 4, with Simulink block dedicated for the Kalman filter. While for the simulation model, the results were very similar to each other, see Figure 6, more distinct differences between two implementations of the Kalman filter appear in experiments on a real plant, see Figure 7, the detail is in Figure 8. Comparison of the nonlinear model simulation results and those from the real plant are in Figure 9.

Simulations for various models, controller parameters and settings of the Kalman filter provide wide possibilities to study the influence of individual control-loop parts on the results. Real plant experiments contribute to bridge the gap between control theory and practice.

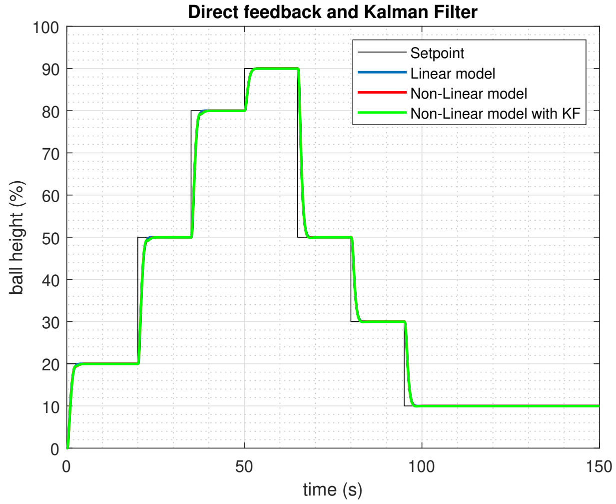

Fig. 5. Comparison of simulation step responses for closed loop with direct state feedback versus step responses for the state feedback with states estimated by the Kalman filter.

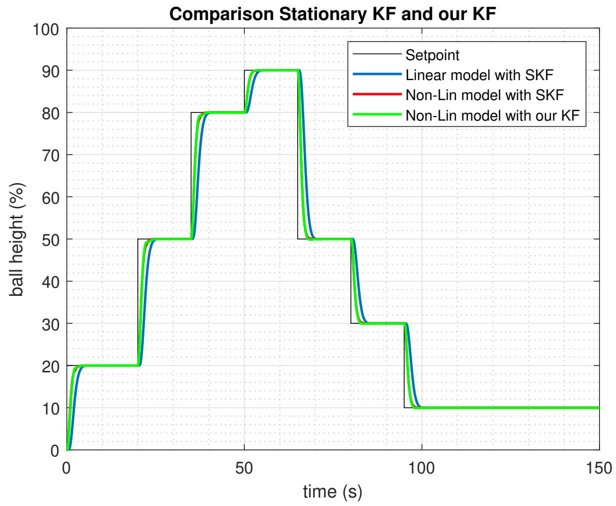

Fig. 6. Comparison of simulation results for two implementations of the Kalman filter.

7. CONCLUSION

Presented material aims at advanced topics of control course and their practical use and implementation on real plants. The proposed content has a direct potential to include controller design of different levels of difficulty. More advanced - pole region approach can be considered for the pole placement controller design with the deeper theoretical background or with the use of already existing computational support (solver). The described implementation of the Kalman filter provides insight into its performance and enables to tune its parameters and directly

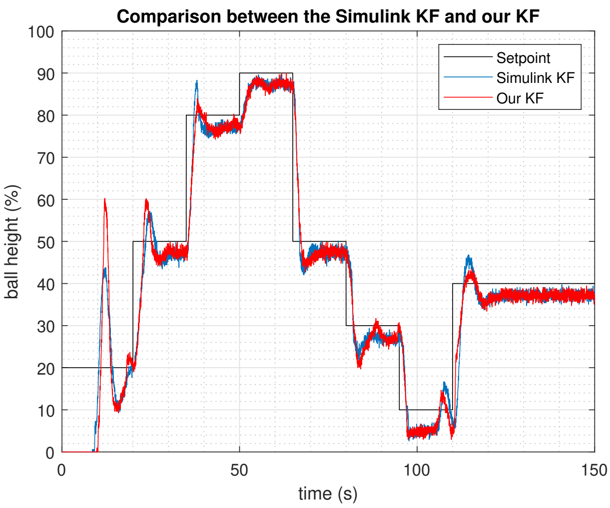

Fig. 7. Comparison of two implementations of Kalman filter on Floatshield plant.

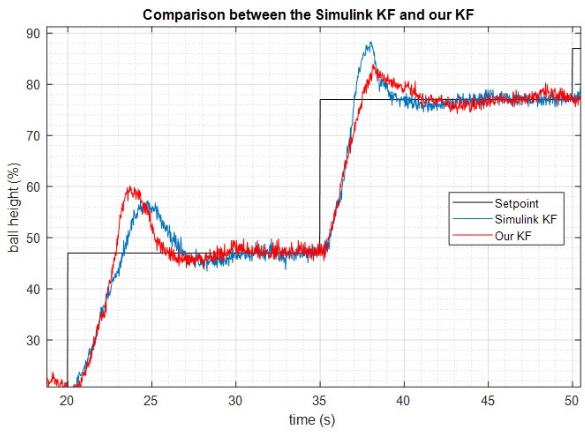

Fig. 8. Comparison of two implementations of Kalman filter on Floatshield plant: detail.

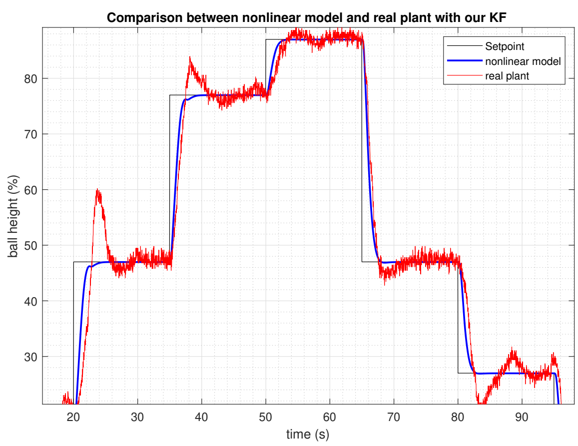

Fig. 9. Comparison of closed-loop responses with our Kalman filter for nonlinear model and real plant.

follow their impact which can contribute to its better understanding. The considered combination of modelling, simulation, theoretical control design and experiments on real device can be covered by 3 to 5 laboratory exercises, depending on the student prerequisites and the scope and depth of the tasks involved.

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REFERENCES

Becker, A. (2019). Online kalman filter tutorial. *URL: https://www. kalmanfilter. net/default. aspx Accessed.*

Bojan-Dragos, C.A., Precup, S.P.R.E., Hughiet, S.H.E., and Szedlak-Stinean, A.I. (2016). State feedback and proportional-integral-derivative control of a magnetic levitation system. In *2016 IEEE 14th International Symposium on Intelligent Systems and Informatics (SISY)*, 111–116.

Bomfim, M., II, E.L., Monteiro, N., and Dias, A. (2021). A novel hybrid (pid+ mrac) adaptive controller for an air levitation system. *IEEE Latin America Transactions*, 19(8), 1400–1409.

Chacon, J., Saenz, J., la Torre, L.D., Diaz, J., and Esquembre, F. (2017). Design of a low-cost air levitation system for teaching control engineering. *Sensors*, 17(10), 2321.

Chacón, J., Vargas, H., Dormido, S., and Sanchez, J. (2018). Experimental study of nonlinear pid controllers in an air levitation system. *IFAC-PapersOnLine*, 51, 304–309.

Chaos, D., Chacón, J., Aranda-Escolástico, E., and Dormido, S. (2020). Robust switched control of an air levitation system with minimum sensing. *ISA transactions*, 96, 327–336.

Chołodowicz, E. and Orłowski, P. (2017). Low-cost air levitation laboratory stand using matlab/simulink and arduino. *Pomiary Automatyka Robotyka*, 21, 33–39.

Espí, J., Castello, J., and Gil, R.G. (2017). Air flow levitation system for in class and remote learning of control systems. *The International journal of engineering education*, 33(1), 74–83.

Hypiusova, M. and Rosinova, D. (2021). Discrete-time pole-region robust controller for magnetic levitation plant. *Symmetry*, 13(1).

Lewis, F., Vrabie, D., and Syrmos, V. (2012). *Optimal control*. John Wiley & Sons.

Pham, T.P., Nguyen, D.Q., Vo, K.T., Hoang, D., et al. (2022). Plc-based pid control system design for an air levitation system. In *2022 7th National Scientific Conference on Applying New Technology in Green Buildings (ATiGB)*, 83–87.

Rosinová, D. and Schwarz, R. (2023). Teaching state-space control methods using a floatshield laboratory plant. In *2023 24th International Conference on Process Control (PC)*, 261–266.

Takács, G. (2022). https://github.com/gergelytakacs/automationshield/wiki/floatshield.

Takács, G., Chmurčiak, P., Gulan, M., Mikuláš, E., Kulhánek, J., Penzinger, G., Vdoleček, M., Podbielančík, M., Lučan, M., Šálka, P., et al. (2020). Floatshield: An open source air levitation device for control engineering education. *IFAC-PapersOnLine*, 53(2), 17288–17295.