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A Generalized Anti-windup Approach for Internal Model Control ⋆

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Abstract: In this paper, we develop a generalized anti-windup synthesis procedure for saturating internal model control (IMC). Within this framework, the anti-windup design problem reduces to a search over a class of left coprime factorization of the linear internal model controller. An appropriate anti-windup is constructed by solving a set of linear matrix inequalities. As compared to other static anti-windup constructions in the literature, the IMC anti-windup is always feasible with guaranteed closed-loop stability and performance. Using a simulation example, we benchmark the behaviour of the constructed anti-windup with existing state-of-the-art methods.

Keywords: Anti-windup, Internal Model Control, Constrained Control.

1. INTRODUCTION

Internal model control (IMC) is a widely studied control technique [1] and a particular case of the celebrated Youla parameterization of all stabilizing controllers [2]. The popularity of IMC derives from its inherent robust stability properties and ease of design [3, 4]. However, for systems with saturating inputs, IMC can be poor-performing despite any stability guarantees [5, 6]. This performance degradation is now well-known to be caused by the windup phenomenon and has attracted considerable attention during the last few decades [7, 8, 9].

The conventional IMC control structure [10], although not intended to be an anti-windup scheme, possesses some inherent anti-windup properties and has served as a baseline example for comparing other anti-windup schemes in the literature [3]. In order to boost the anti-windup performance of the IMC, several enhancements have been suggested for the conventional IMC structure [5, 12, 14]. Most of the enhancements are in the form of the modified IMC structure [6] where the IMC controller is split into two with one-half wrapped around the input saturation. This IMC controller split, usually based on heuristics, allows for a trade-off between performance and stability [5, 6].

In this work, we develop a systematic approach for IMC anti-windup design that preserves the stability properties of the conventional IMC and guarantees some level of performance defined in terms of quick recovery of linear performance after periods of input saturation. This approach formalizes the modified IMC anti-windup design [6] in terms of the left coprime factorization of the linear IMC controller. This characterization, although related to other coprime factorization based anti-windup schemes (e.g. [3, 16]), has largely been omitted in the anti-windup literature. In this work, we seek to bridge this gap to deliver an anti-windup that combines the ease of modern anti-windup construction with the benefits of the IMC structure.

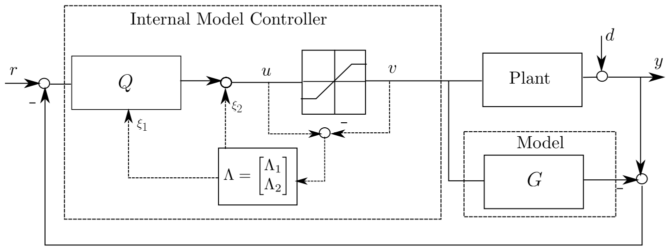

Fig. 1. The Generalized IMC Anti-windup Structure

2. THE ANTI-WINDUP PROBLEM DEFINITION

We motivate the anti-windup design problem using the IMC structure.

2.1 IMC Anti-windup Construction

We consider the anti-windup set-up of Fig.1 comprising a linear time-invariant system

$$G(s) \sim \begin{cases} \dot{x}_p = A_p x_p + B_p v, \\ y_p = C_p x_p + D_p v, \end{cases} \tag{1}$$

a nominal IMC controller

$$Q(s) \sim \begin{cases} \dot{x}_q = A_q x_q + B_q w, \\ y_q = C_q x_q + D_q w, \end{cases} \tag{2}$$

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and the augmentation

$$\xi_1 = \Lambda_1(u - v),\ \xi_2 = \Lambda_2(u - v) \quad (3)$$

where Λ_1 and Λ_2 are static anti-windup gains. The corrective signals ξ_1 and ξ_2 are triggered by a saturating input nonlinearity $v = \mathrm{sat}(\mathrm{u})$:

$$\mathrm{sat}(u) = \begin{cases} \mathrm{sat}_1(u_1), \\ \vdots \\ \mathrm{sat}_n(u_n) \end{cases}, \mathrm{sat}_i(u_i) = \begin{cases} \overline{u}_i, u_i \geq \overline{u}_i, \\ u_i, \underline{u}_i < u_i < \overline{u}_i, \\ \underline{u}_i, u_i \leq \underline{u}_i, \end{cases} \quad (4)$$

where $\underline{u}_i$ and $\overline{u}_i$ are respectively the lower and upper bounds on the control input u_i. The generalized IMC anti-windup structure is obtained by augmenting the nominal IMC controller (2) with (3) as

$$\begin{cases} \dot{x}_q = A_q x_q + B_q w + \xi_1, \\ u = C_q x_q + D_q w + \xi_2. \end{cases} \quad (5)$$

Assuming a perfect model, the interconnection with $u = y_q$ and $w = r - d$ recovers the conventional IMC structure that is generally considered a baseline scheme for comparing several emerging anti-windup approaches. No doubt, this structure possesses some anti-windup properties as any saturating effect at the plant input is fed through the plant model to the controller. However, if the plant has slow modes, lightly damped modes or non-minimum phase zeros, the closed-loop response during input saturations can be very poor [6]. The additional degree of design freedom through the static gains Λ_1 and Λ_2 can be exploited to enhance the performance of the conventional IMC anti-windup.

2.2 Coprime Factorization Parameterization

Incorporating the anti-windup block into the nominal closed-loop, the anti-windup compensation (5) can be parameterized in terms of the left coprime factorization of the IMC controller Q. From the augmented controller:

$$\begin{cases} \dot{x}_q = (A_q + H_1 C_q)x_q + (B_q + H_1 D_q)w - H_1 v \\ u = H_2 C_q x_q + H_2 D_q w + (I - H_2)v \end{cases} \quad (6)$$

we have

$$[Q_1\ Q_2] \sim \begin{cases} \dot{x}_q = (A_q + H_1 C_q)x_q + (B_q + H_1 D_q)w + H_1 v \\ u = H_2 C_q x_q + H_2 D_q w + (H_2 - 1)v \end{cases}$$

where $H_1 = \Lambda_1(I - \Lambda_2)^{-1}$ and $H_2 = (I - \Lambda_2)^{-1}$. It is straightforward to verify that

$$Q = (I + Q_2)^{-1} Q_1 \quad (7)$$

holds and the anti-windup construction can be interpreted as that of splitting Q into two factors Q_1 and Q_2 through appropriate choice of matrices H_1 and H_2.

It is insightful to note that earlier works on IMC-based anti-windup construction focused on splitting Q into two Q_1 and Q_2 using various heuristics (see, e.g., [5, 6]). An attempt was made in [11] to unify these approaches using a static anti-windup gain. In this work, we formalize such splits using the framework of coprime factorization and we construct an appropriate split by optimizing over a set of linear matrix inequalities. While the concept of coprime factorization has largely been exploited in anti-windup synthesis, see, e.g., [16, 18] for anti-windup parameterization in terms of the left coprime factorization of the nominal feedback controller and [3, 20] for parameterization in terms of the right coprime factorization of the nominal plant, to the best of our knowledge, this is the first time an anti-windup will be characterized in terms of the coprime factorization of the IMC controller Q. We comment on the connection of our approach to these existing schemes under related works.

2.3 Decoupling Architecture of the IMC Anti-windup

The IMC anti-windup structure can be decoupled into a linear part and a perturbed part. We write the closed-loop equations for the IMC anti-windup as:

$$\begin{aligned} y &= Gv + d, \\ u &= Q_1 e - Q_2 v. \end{aligned} \quad (8)$$

Using the relationship between saturation and dead-zone non-linearities: $q = \mathrm{dz}(u) = u - \mathrm{sat}(u)$, (8) reduces to

$$\begin{aligned} y &= GQw - G(I + Q_2)^{-1} q + d \\ u &= Qw + (I + Q_2)^{-1} Q_2 q \end{aligned} \quad (9)$$

which can be interpreted at the perturbation of the nominal closed-loop as

$$\begin{aligned} y &= y_{lin} - G(I + Q_2)^{-1} q \\ u &= u_{lin} + (I + Q_2)^{-1} Q_2 q \end{aligned} \quad (10)$$

where $y_{lin} = GQw + d$, and $u_{lin} = Qw$ describe the nominal closed-loop and

$$\begin{cases} \begin{bmatrix} G(I + Q_2) \\ (I + Q_2)^{-1} Q_2 \end{bmatrix} \sim \\ \dot{x} = \begin{bmatrix} A_q & 0 \\ -B_p C_q & A_p \end{bmatrix} x + \begin{bmatrix} \Lambda_1 \\ B_p(I - \Lambda_2) \end{bmatrix} v \\ \begin{bmatrix} y_d \\ u_d \end{bmatrix} = \begin{bmatrix} -D_p C_q & C_p \\ C_q & 0 \end{bmatrix} x_p + \begin{bmatrix} D_p(I - \Lambda_2) \\ \Lambda_2 \end{bmatrix} v. \end{cases} \quad (11)$$

So the anti-windup design objective can be formulated as that of making (11) small in some sense such that y stays close to y_{lin}, and u close to u_{lin} during saturation and recovers linear behaviour during small signal operation.

The augmented IMC controller can also be decoupled into a linear part and a non-linear part. Rewriting (6), we have the interconnection of

$$\begin{cases} \dot{x}_q = (A_q + H_1 C_q)x_q + (B_q + H_1 D_q)w - H_1 v \\ u_o = C_q x_q + D_q w \end{cases} \quad (12)$$

and

$$\begin{cases} v = \mathrm{sat}(u) \\ u = H_2 u_o + (I - H_2)v \end{cases} \quad (13)$$

where (13) describes an algebraic loop that can be exploited to enhance the anti-windup performance [16]. The decoupling between H_1 and H_2 in (12) and (13) is such that the anti-windup design process can be decoupled into finding an H_1 such that (12) is exponentially stable and then finding H_2 such that the algebraic loop is well-posed and induces acceptable performance in closed-loop. Procedure for constructing an appropriate H_2 is well established in the literature (see, e.g., [13] for heuristics and [17] for implementation).

Here, we take advantage of the decoupling in (10) to construct an appropriate anti-windup gain (Λ_1, Λ_2) that effectively factorizes Q into (Q_1, Q_2), obviating the need for heuristics (cf. [6]) and offering both stability and L_2 performance guarantees.

2.4 Related Anti-windup Schemes

We discuss related anti-windup schemes that make use of internal model within the anti-windup construction similarly to the IMC. We comment on the connections between these schemes and our proposed IMC anti-windup.

Linear Model Recovery Anti-windup [8, 19] Suppose the nominal unity feedback controller is given by

$$C(s) \sim \begin{cases} \dot{x}_c = A_c x_c + B_c(r - y_p), \\ y_c = C_c x_c + D_q(r - y_p), \end{cases} \tag{14}$$

the linear Model Recovery Anti-windup (MRAW) is based on the augmentation:

$$\begin{cases} \dot{x}_{aw} = A_p x_{aw} + B_p(\text{sat}(y_c - \xi_1) - y_c) \\ \xi_1 = K x_{aw} + L(\text{sat}(y_c - \xi_1) - y_c) \\ \xi_2 = C_p x_{aw} + D_p(\text{sat}(y_c - \xi_1) - y_c) \end{cases} \tag{15}$$

where ξ_1 and ξ_2 are signals injected at the output and input of the nominal feedback controller respectively. The matrices K and L are anti-windup gains to be synthesized. By writing $F = (I - L)^{-1}K$ and $E = (I - L)^{-1}$, the linear MRAW can be expressed as:

$$\begin{cases} \dot{x}_{aw} = (A_p + B_p F)x_{aw} + B_p E q \\ \xi_1 = F x_{aw} + (E - I)q \\ \xi_2 = (C_p + D_p F)x_{aw} + D_p E q. \end{cases} \tag{16}$$

This is the right coprime factorization of the plant $G = NM^{-1}$ with factors:

$$\begin{bmatrix} M \\ N \end{bmatrix} \sim \begin{cases} \dot{x}_{aw} = (A_p + B_p F)x_{aw} + B_p E q \\ \xi_1 = F x_{aw} + E q \\ \xi_2 = (C_p + D_p F)x_{aw} + D_p E q. \end{cases} \tag{17}$$

Observe that setting by $(K = 0,\ L = 0)$ and hence $(F = 0,\ E = I)$, the linear MRAW recovers the conventional IMC anti-windup structure. The linear MRAW anti-windup is an enhancement of the conventional IMC through an additional degree of freedom offered by anti-windup gain $(K,\ L)$. This gain plays a similar role to the IMC anti-windup gain $(\Lambda_1,\ \Lambda_2)$ of the proposed scheme.

Weston-Postlethwaite Anti-windup [20] The Weston-Postlethwaite anti-windup (WPA) is parameterized in terms of filter $M(s)$ and the linear plant dynamics. Using the decoupling architecture, the WPA can be expressed as:

$$\begin{aligned} y &= y_{lin} - GMq \\ u &= u_{lin} + (M - I)q \end{aligned} \tag{18}$$

where $y_{lin} = Gu_{lin} + d$ and $u_{lin} = C(r - y_{lin})$ describe the unconstrained closed loop. It therefore follows that choosing $M = (I + Q_2)^{-1}$, the WPA recovers the proposed IMC anti-windup. While there are other options for filter $M(s)$, the most suitable choice is based the right coprime factorization of the plant [3, 21]. Using the right coprime factorization in (17), the compensator reduces to (16). It is then obvious that the WPA is equivalent to the linear MRAW scheme but for the way gains F and E are constructed. Observe that for the choice $M(s) = I$, the WPA recovers the conventional IMC anti-windup.

Embedded IMC Anti-windup [22] The embedded IMC scheme is based on the augmentation:

$$\begin{cases} \dot{x}_{aw} = A_p x_{aw} + B_p q, \\ y_{im} = C_p x_{aw} + D_p q, \\ \xi_1 = \Lambda_1 q,\ \xi_2 = \Lambda_2 q \end{cases} \tag{19}$$

where y_{im}, ξ_1 and ξ_2 are signals injected at the input, states and output of the nominal feedback controller respectively. Observe that (19) has an explicit copy of the nominal plant in addition to static anti-windup gains Λ_1 and Λ_2. For the special case where $(\Lambda_1 = 0,\ \Lambda_2 = 0)$, the embedded IMC scheme recovers the conventional IMC anti-windup. While the embedded IMC anti-windup is closely related to our proposed IMC anti-windup, the performance optimization in [22] is based on a global L_2 gain that has nothing to do with the recovery of linear performance after periods of input saturation.

3. IMC ANTI-WINDUP ANALYSIS AND SYNTHESIS

3.1 Stability and Performance Analyses

Anti-windup design philosophy is based on the assumption that the nominal closed-loop is already designed to be stable with acceptable performance. For the IMC-anti-windup, the nominal closed-loop can be expressed as

$$\begin{bmatrix} y_{lin} \\ u_{lin} \end{bmatrix} = \begin{bmatrix} G & I - GQ \\ Q & -Q \end{bmatrix} \begin{bmatrix} r \\ d \end{bmatrix} \tag{20}$$

where we have assumed a perfect model. It thus follows that internal stability of the nominal IMC closed-loop requires the exponential stability of both the plant G, and the IMC controller Q (see, e.g., [23, Lemma 4.6]). Therefore, for our subsequent discussions, we assume that the nominal closed-loop, in the absence of input saturation, is already well-designed in that both G and Q are exponentially stable and their interconnection induces an acceptable performance.

Now considering the closed-loop equation in (9) and its decoupled form in (10), we only need to focus on the perturbation terms in order to enforce closed-loop stability during episodes of input saturation and to recover linear or nominal performance after such episodes. Note that the linear perturbation terms $G(I + Q_2)^{-1}$ and $(I + Q_2)^{-1}Q_2$ are both exponentially stable since G and Q are both assumed to be exponentially stable (see the system matrix in (11)). So closed-loop stability reduces to enforcing stability of the nonlinear loop comprising the interconnection of $u_d = (I + Q_2)^{-1}Q_2 q$ and the deadzone nonlinearity $q =$ u via $u = u_{lim} + u_d$. Following standard anti-windup design approach, this task can be accomplished by invoking multivariable circle criterion, taking advantage of the sector-bound property of the deadzone. For performance, since we are interested in the quick recovery of nominal performance after periods of control input incursion into the saturating region, we focus on minimizing the L_2 gain of the map from u_{lin} to y_d.

3.2 IMC-based Anti-windup Synthesis

We develop a procedure for constructing the IMC-based anti-windup using Linear Matrix Inequalities (LMIs).

LMI-Based Synthesis

$$\begin{bmatrix} YA^T + AY & B_oU + B_\xi X + YC_u^T & 0 & YC_y^T \\ UB_o^T + X^T B_\xi^T + C_u Y & D_\alpha X + X^T D_\alpha^T - 2U & I & UD_p^T + X^T D_\xi^T \\ 0 & I & -\gamma I & 0 \\ C_y Y & D_p U + D_\xi X & 0 & -\gamma I \end{bmatrix} < 0\,, \qquad \mathbf{(21)}$$

Theorem 1. Given a nominal plant G and a nominal IMC controller Q, both of which are exponentially stable. Suppose there exists matrices $Y = Y^T > 0$, $U = U^T > 0$ (diagonal), X (arbitrary) and a gain $\gamma > 0$ such that the LMI condition (21) holds: where the constant matrices are given as:

$$A = \begin{bmatrix} A_q & 0 \\ -B_pC_q & A_p \end{bmatrix}, B_o = \begin{bmatrix} 0 \\ B_p \end{bmatrix}, B_\xi = \begin{bmatrix} I & 0 \\ 0 & -B_p \end{bmatrix}$$
$$\begin{bmatrix} C_y \\ C_u \end{bmatrix} = \begin{bmatrix} -D_pC_q & C_p \\ C_q & 0 \end{bmatrix}, \text{ and } \begin{bmatrix} D_\xi \\ D_\alpha \end{bmatrix} = \begin{bmatrix} 0 & -D_p \\ 0 & I \end{bmatrix}. \qquad (22)$$

Then the IMC anti-windup guarantees closed-loop stability and ensures that the L_2 gain from u_{lin} to y_d is less than γ. Moreover the anti-windup gain $\Lambda = \begin{bmatrix} \Lambda_1^T & \Lambda_2^T \end{bmatrix}^T$ is recovered as $\Lambda = XU^{-1}$ where X and U are feasible solutions of (21).

Proof. The proof is standard (cf. [3, 12]) and involves showing that (21) enforces both quadratic stability and L_2 gain from u_{lin} to y_d through:

$$\dot{V}(x) + 2q^T W(q-u) + y_d^T y_d - \gamma^2 u_{lin}^T u_{lin} < 0 \qquad (23)$$

where $V = x^T P x > 0$ enforces quadratic stability of the anti-windup loop, $y_d^T y_d - \gamma^2 u_{lin}^T u_{lin} < 0$ enforces the L_2 gain from u_{lin} to y_d to be less than γ and $q^T W(u - q) > 0$ with diagonal $W > 0$ is the sector characterization of the input nonlinearity. Substituting for $X = \Lambda U$ in (21) followed by a sequence of congruence transformation using $\mathrm{diag}(P, W, I, I)$ where $P = Y^{-1}$ and $W = U^{-1}$ and Schur complement transformation yields the following inequality:

$$\begin{bmatrix} A^T P + PA & P(B_o + B_\xi\Lambda) + C_u^T W & 0 \\ (B_o + B_\xi\Lambda)^T P + WC_u & WD_\alpha^T \Lambda + \Lambda^T D_\alpha^T W - 2W & W \\ 0 & W & -\gamma^2 I \end{bmatrix}$$
$$+ \begin{bmatrix} C_y^T \\ (D_p + D_\xi\Lambda)^T \\ 0 \end{bmatrix} [C_y \;\; (D_p + D_\xi\Lambda) \;\; 0] < 0. \qquad (24)$$

It then follows that multiplying (24) on the left by $\begin{bmatrix} x^T & q^T & u_{lim}^T \end{bmatrix}$ and on the right by $\begin{bmatrix} x^T & q^T & u_{lim}^T \end{bmatrix}^T$ yields (23).

Remark 2. Note that while the anti-windup construction in Theorem 1 is for a static anti-windup gain Λ, the resulting IMC anti-windup is dynamic. There is always a choice of Λ that ensures the existence of an anti-windup contrary to the static anti-windup construction in ([15, 16]) where an anti-windup is not guaranteed to always exist. For the special case of $\Lambda = 0$, the proposed anti-windup recovers the conventional IMC anti-windup which is a trivial solution to (21).

We comment on the feasibility of the synthesis LMI (21).

Feasiblity of the LMI Solution

Corollary 3. The synthesizing LMI problem (21) is feasible if and only if there exists a solution (P, γ) to the following LMI conditions:

$$\begin{bmatrix} A_p^T P_{22} + P_{22}A_p & P_{22}B_p & C_p^T \\ B_p^T P_{22} & -\gamma I & D_p^T \\ C_p & D_p & -\gamma I \end{bmatrix} < 0, \qquad (25)$$

$$\begin{bmatrix} A^T P + PA & C_y^T \\ C_y & -\gamma I \end{bmatrix} < 0 \qquad (26)$$

where $Y = P^{-1}$ and $P = P^T = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} > 0$.

Proof. The proof follows from applying the projection lemma (e.g., see, [23, Chapter 12]) to (24). Re-writing (24) as

$$\begin{bmatrix} PB_\xi \\ WD_\alpha \\ 0 \\ D_\xi \end{bmatrix} \Lambda \,[0 \;\; I \;\; 0 \;\; 0] + \begin{bmatrix} 0 \\ I \\ 0 \\ 0 \end{bmatrix} \Lambda^T \begin{bmatrix} B_\xi^T P & D_\alpha^T W & 0 & D_\xi^T \end{bmatrix}$$
$$+ \begin{bmatrix} A^T P + PA & PB_o + C_u^T W & 0 & C_y^T \\ B_o^T P + WC_u & -2W & W & D_p^T \\ 0 & W & -\gamma I & 0 \\ C_y & D_p & 0 & -\gamma I \end{bmatrix} < 0. \qquad (27)$$

First, we construct appropriate bases for the null spaces of $[0 \;\; I \;\; 0 \;\; 0]$ and $\begin{bmatrix} B_\xi^T & D_\alpha^T & 0 & D_\xi^T \end{bmatrix}$ respectively as

$$\mathcal{W}_R = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \text{ and } \mathcal{W}_L = \begin{bmatrix} 0 & 0 & 0 \\ I & 0 & 0 \\ B_p^T & 0 & D_p^T \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \qquad (28)$$

Next, pre- and post multiplying (27) by $\mathcal{W}_R^T$ and $\mathcal{W}_R$ respectively yields (26). Now applying congruence transformation with $\mathrm{diag}(Y, U, I, I)$ to (27) followed by pre-and post multiplication by $\mathcal{W}_L^T$ and $\mathcal{W}_L$ respectively yields

$$\begin{bmatrix} A_pY_{22} + Y_{22}A_p^T & B_p & Y_{22}C_p \\ B_p^T & -\gamma I & D_p^T \\ C_pY_{22} & D_p & -\gamma I \end{bmatrix} < 0. \qquad (29)$$

Then a congruence transformation with $\mathrm{diag}(P_{22}, I, I)$ yields (25). Then by the projection lemma, (27) is solvable for Λ if and only if (25) and (26) hold.

Remark 4. Conditions (25) and (26) have system theoretical interpretations. In particular, (25) is equivalent to the bounded real lemma that constrains γ to be no less than the infinity norm of the open-loop plant. Since A_p is Hurwitz, there is a $P_{22} = P_{22}^T > 0$ satisfying $A_P^T P_{22} + P_{22}A_p < 0$. So, (25) is always feasible and the infinity norm of the open-loop plant provides a lower bound for γ. Similarly, the third term in (27) constrains γ to be no less than the L_2 gain of the nonlinear loop corresponding to the conventional IMC. Since A_P and A_q are both assumed to be Hurwitz, A is Hurwitz and there is a $P = P^T > 0$ satisfying $A^T P + PA < 0$. So for $\Lambda = 0$, and $W > 0$, (27) is always feasible for sufficiently large γ. Finally, since (26) is a principal submatrix of (27), (26) is always feasible. So by Corollary (3), the synthesizing LMI (21) is always solvable for Λ.

Remark 5. The instantaneous feed-through via Λ_2 around the nonlinearity in Fig. 1 means that the IMC anti-windup must deal with the issue of algebraic loop. While the (2,2) block of the synthesizing LMI, alternatively expressed as $\Lambda_2 U + U\Lambda_2^T - 2U < 0$, guarantees the well-posedness of such algebraic loops, the LMI solutions are often such that the algebraic loop is close to the margin of being ill-posed or that $H_2 = (I - \Lambda_2)^{-1}$ is close to being singular. To avoid ill-conditioning, an additional constraint in the form:

$$D_\alpha X + X^T D_\alpha - 2(1-\rho)U < 0 \quad (30)$$

where $0 < \rho < 1$ is usually appended to the synthesising LMI to enforce stronger well-posedness (see, e.g., [8, 17]).

4. SIMULATION RESULTS

We illustrate the effectiveness of the generalized IMC anti-windup when compared with other related anti-windup schemes that incorporate an internal model in their anti-windup compensation. We consider a multivariable example taken from [6, 16] with the description:

$$\begin{cases} \dot{x}_p = \begin{bmatrix} -0.01 & 0 \\ 0 & -0.01 \end{bmatrix} x_p + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} u\,, \\ y_p = \begin{bmatrix} 0.4 & -0.5 \\ -0.3 & 0.4 \end{bmatrix} x_p\,. \end{cases} \quad (31)$$

The unity feedback controller for the linear plant is [16]:

$$\begin{cases} \dot{x}_c = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} x_c + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} (r - y_p)\,, \\ y_c = \begin{bmatrix} 0.020 & 0.025 \\ 0.015 & 0.020 \end{bmatrix} x_c + \begin{bmatrix} 2.0 & 2.5 \\ 1.5 & 2.0 \end{bmatrix} (r - y_p). \end{cases} \quad (32)$$

and the IMC controller designed for a step input is [6]:

$$\begin{cases} \dot{x}_q = \begin{bmatrix} -0.05 & 0 \\ 0 & -0.05 \end{bmatrix} x_q + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} w\,, \\ u = \begin{bmatrix} -0.08 & -0.10 \\ -0.06 & -0.08 \end{bmatrix} x_q + \begin{bmatrix} 2.0 & 2.5 \\ 1.5 & 2.0 \end{bmatrix} w\,. \end{cases} \quad (33)$$

The inputs to the plant are constrained such that $|u_i| \leq 1, i = 1, 2$, and a set-point change $r = [0.63, 0.79]^T$ is applied at time $t = 0$. For this example, the conventional IMC provides a baseline anti-windup with stability guarantee but very poor performance [6]. Minimizing γ subject to the LMI constraint in (21) and the strong well-posedness constraint in (30) (with ρ chosen as 0.05) gives the anti-windup gains (Λ_1, Λ_2) respectively as:

$$\begin{bmatrix} -34.3696 & -26.2304 \\ -46.8557 & -37.0488 \end{bmatrix}, \begin{bmatrix} -371.7663 & -290.9602 \\ -291.3070 & -226.4984 \end{bmatrix} \quad (34)$$

with an L_2 gain value of 81.2311. Figs. 2 through 5 show both the output and the control input responses when the proposed IMC anti-windup is compared to other anti-windup schemes discussed in Section 2.4.

Observe that as compared to the two degrees of freedom available through Λ_1 and Λ_2 in the current design, the approach in [11] is based on one degree of freedom and hence its inferior transient response. For the linear MRAW scheme, we follow [8, Algorithm 12] which is based on a linear-quadratic performance measure, to construct K and L as in (15). For the WPA scheme, we follow [3] which sets $E = I$ in (17) and hence has no algebraic loop.

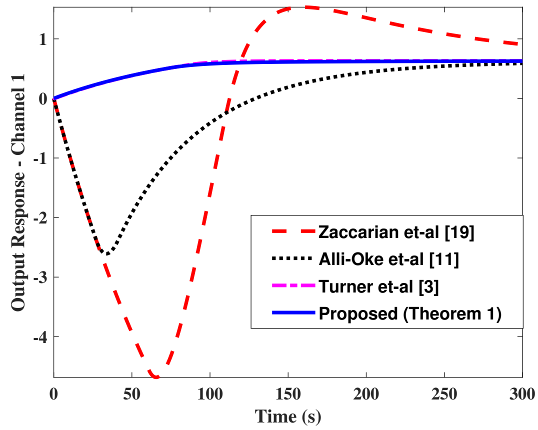

Fig. 2. Channel-1 Output Response using the Proposed IMC Anti-windup benchmarked with [11], [3], and [19].

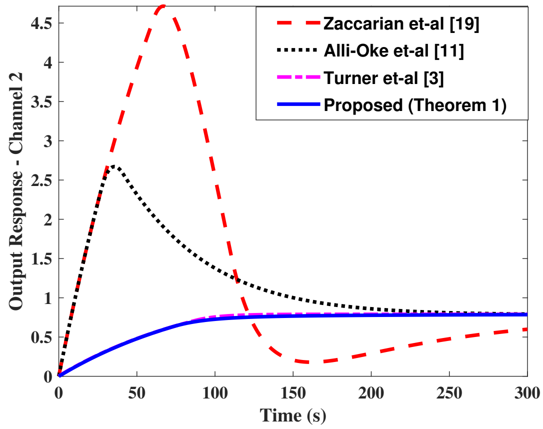

Fig. 3. Channel-2 Output Response using the Proposed IMC Anti-windup benchmarked with [11], [3], and [19].

As shown, the proposed method compares favourably with [3] and outperforms the other schemes during the transient phase. While the recovery of linear performance is as swift as in [3], the proposed IMC anti-windup method provides insights into the operation of the conventional IMC and on how the IMC performance can be greatly enhanced through appropriate left coprime factorization of the nominal IMC controller.

5. CONCLUSION

We have constructed an anti-windup procedure for the internal model control (IMC). The ensuing anti-windup is parameterized in terms of two static gains that define a left coprime factorization of the nominal IMC controller. The method complements existing anti-windup methods that rely on the coprime factorization of either the feedback controller or the plant. Results from a benchmark simulation example showed that the generalized IMC anti-windup can compare favourably with or outperform existing anti-windup techniques.

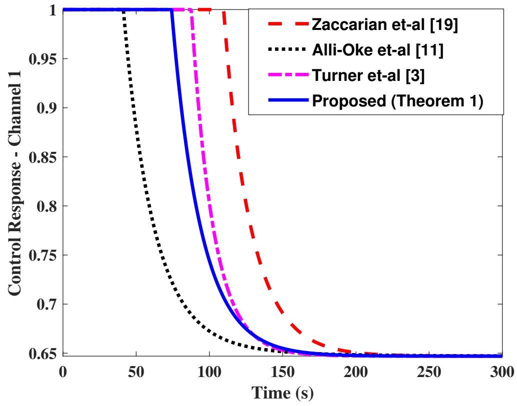

Fig. 4. Channel-1 Input Response using the Proposed IMC Anti-windup benchmarked with [11], [3], and [19].

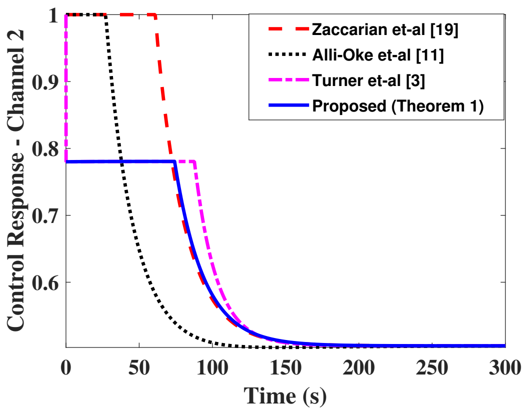

Fig. 5. Channel-2 Input Response using the Proposed IMC Anti-windup benchmarked with [12], [3], and [19].

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