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A Comparison of Two Methods of Adaptive Nonlinear Model Predictive Control

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Abstract: Conventional nonlinear model predictive control (NMPC) relies on an accurate process model. However, real-world systems' models are often imperfect due to parametric variations, modelling errors and additive noise, leading to degraded control performance. This study investigates the effectiveness of two adaptive NMPC methods in solving this problem, namely, linear parameter varying (LPV) model predictive control (LPV-MPC) and model predictive control based on successive linearization (SL-MPC). In addition, an innovative approach for identifying LPV models is proposed and applied to three simulation examples. The identified LPV models gave a very strong fit. Also, simulation results demonstrate that both adaptive predictive NMPC (LPV-MPC and SL-MPC) exhibit performance similar to conventional NMPC and superior to Linear MPC. Notably, the two adaptive predictive controllers offer significantly reduced computational time compared to conventional NMPC.

Keywords: Adaptive model predictive control, linear parameter varying (LPV) model, nonlinear predictive control, Linear time varying (LTV) model.

1. INTRODUCTION

Industrial process control faces many challenging problems among which are uncertainties in process model. The conventional NMPC (Mayne, 2014) which is the standard for addressing these problems, rely on an accurate dynamic model. However, real-world systems are often subject to uncertainties, parameter variations, and frequent disturbances, leading to suboptimal control performance and, in some cases, instability. Thus, adaptive NMPC (Adetola *et al.,* 2009) presents an appealing control solution.

An adaptive NMPC typically integrates a parameter updating algorithm with the NMPC algorithm. Different adaptive NMPC techniques primarily vary based on the type of identification method used in updating the model parameters in real-time. Various types of adaptive NMPC have been proposed in previous research. Adetola *et al.* (2009) combined a recursive least square (RLS) parameter adjustment mechanism with a robust stabilizing NMPC, yielding interesting results. However, this approach often leads to non-convexity of the optimization problem. Kohler *et al*. (2019) proposed a combination of set-membership (SM) identification method and tube MPC with guaranteed recursive feasibility and robust constraint satisfaction. However, its extension to large-scale nonlinear system is not straightforward.

Other popular methods for designing adaptive NMPC involves the use of extended Kalman filter (EKF) (Lee and Ricker, 1994) and moving horizon estimation (MHE) (Huang *et al.,* 2010) for state and parameter estimation. Also, some researchers combined learning model such as neural network with NMPC to achieve adaptation (Akpan and Hassapis, 2011). However, these approaches often suffer from computational complexity.

Two other common techniques that explore linear model structure to achieve adaptation are linear parameter varying model based predictive control (LPV-MPC) and model predictive control based on successive linearization (SL-MPC). The review journal by Morato *et al.* (2020) surveyed the different methods available for LPV-MPC design, while Cox and Toth (2021) proposed a sub-space identification method for LPV models. Bamimore (2023) proposed a gap-metric based method for computing and designing LPV-MPC. SL-MPC, on the other hand, was first proposed by Ricker *et al.* (2009). Seki (2004) reported its successful application to industrial grade polymerization reactor, while Bamimore *et al.* (2011) reported its application to an experimental three-tank system.

There are still many open research questions in adaptive NMPC. For instance, there is a need for robust data-driven identification method for LPV models. There is also the need to improve on numerical efficiency of LPV model identification algorithms in order to reduce computational load. This present study seeks to bridge these research gaps. Thus, the main contribution of this study is the proposition of an innovative method of selecting the starting point for LPV model identification and subsequent determination of optimal model parameters using an optimization method. In addition, the present study also compares the performance of SL-MPC with LPV-MPC in order to highlight their merits and demerits.

The structure of the paper is as follows: Section 2 presents the problem statement of an adaptive NMPC; Section 3 proposes the theory and the algorithm for implementing LPV-MPC and SL-MPC. Simulation results are provided in Section 4, while Section 5 concludes the study.

2. PROBLEM STATEMENT

2.1 Process system and system model

Consider that the process system to be controlled can be described by:

$$x_{k+1}^p = f_p(x_k^p, u_k, w_k) \quad (1)$$

$$y_k^p = g_p(x_k^p, v_k) \quad (2)$$

where $x_k^p \in \mathcal{X} \subseteq \mathbb{R}^{n_x}$, $u_k \in \mathcal{U} \subseteq \mathbb{R}^{n_u}$ and $y_k^p \in \mathcal{Y} \subseteq \mathbb{R}^{n_y}$ are vectors of state, control input and controlled variables, respectively. The variables $w_k \in W_x \subseteq \mathbb{R}^{n_w}$ and $v_k \in V_y \subseteq \mathbb{R}^{n_v}$ are vectors of state and output noises, respectively. The control objective is to design a computationally efficient adaptive NMPC which stabilizes the system at an equilibrium state, x_{ss} in the face of process/model mismatch while satisfying the system constraints so that the system output tracks the setpoint:

$$y_k = g(x_{ss}) = r_k \quad (3)$$

To facilitate controller design, the nominal model of the process, Eqs (1) and (2), is often represented as:

$$x_{k+1} = f(x_k, u_k) \quad (4)$$

$$y_k = g(x_k) \quad (5)$$

The linearization of the Eqs (4) and (5) at the nominal operating point (denoted using subscript, ss) usually leads to linear time invariant (LTI) model:

$$x_{k+1} = f(x_{ss}, u_{ss}) + A\delta x_k + B\delta u_k \quad (6)$$

$$y_k = g(x_{ss}) + C\delta x_k \quad (7)$$

where $A = \left.\frac{\partial f}{\partial x_k}\right|_{x_{ss},u_{ss}}$, $B = \left.\frac{\partial f}{\partial x_k}\right|_{x_{ss},u_{ss}}$ and $C = \left.\frac{\partial g}{\partial x_k}\right|_{x_{ss},u_{ss}}$

2.2 Adaptive Nonlinear Model Predictive Control

An Adaptive NMPC is a control strategy that adjusts its prediction model online to adapt to changes in the system dynamics. The basic idea is to use a prediction model of the process to optimize the control moves over a finite time horizon, and then apply only the first control move to the process. This procedure is repeated at each time step. It is formulated as determining the control moves over the prediction horizon, i.e., $U = \{\bar{u}_k, \bar{u}_{k+1}, \dots, \bar{u}_{k+N_p-1}\}$, which minimize the performance index:

$$\min_U \mathcal{J}(x_k, \bar{u}_k) = \sum_{i=0}^{N_p-1} l(e_{k+i}, \bar{u}_{k+i}) + V_f\left(x_{k+N_p}\right) \quad (8)$$

$$l(e_{k+i}, \bar{u}_{k+i}) = \left(\|e_{k+i}\|_{W_y}^2 + \|\Delta\bar{u}_{k+i-1}\|_{W_{\Delta u}}^2\right) \quad (9)$$

$$V_f\left(x_{k+N_p}\right) = \left\|x_{k+N_p}\right\|_{W_P}^2 \quad (10)$$

subject to:

$$x_{k+i+1} = x_{ss} + A_k\delta x_{k+i} + B_k\delta\bar{u}_{k+i} + B_d\delta d_{k+i} + \Delta x_{ss} \quad (11)$$

$$\hat{y}_{k+i} = y_{ss} + C_k\delta x_k + C_d\delta d_{k+i} \quad (12)$$

$$\delta d_{k+i+1} = \delta d_{k+i} \quad (13)$$

$$\Delta\bar{u}_{k+i} = \bar{u}_{k+i} - \bar{u}_{k+i-1} \text{ for } i = 0, \dots, N_p - 1 \quad (14)$$

$$x_k = \hat{x}_k \quad (15)$$

$$x_{k+i} \in \mathcal{X} \text{ for } i = 0, \dots, N_p \quad (16)$$

$$\bar{u}_{k+i} \in \mathcal{U} \text{ for } i = 0, \dots, N_p - 1 \quad (17)$$

$$\Delta\bar{u}_{k+i} \in \Delta\mathcal{U} \text{ for } i = 0, \dots, N_p - 1 \quad (18)$$

$$z_{k+N_p} \in \Omega \quad (19)$$

where $e_{k+i} = r_{k+i} - y_{k+i}$, N_p represents the prediction horizon; W_y and $W_{\Delta u}$ denote output and incremental input weighting matrices, respectively; r stands for the setpoint; W_p represents the terminal weighting matrix; $\|x\|_W^2$ is defined as $x^T W x$. The variable d stands for an augmented disturbance variables which provide the predictive controller with an integral action. The delta (δ) symbol indicates variables in their deviation form. The variables, x_{ss}, Δz_{ss}, u_{ss}, and y_{ss} are the nominal values of the state, incremental state, manipulated input and output, respectively. These variables, x_{ss}, u_{ss}, and y_{ss} are updated online as new measurements are available while the variable Δx_{ss} is calculated as:

$$\Delta x_{ss} = (A_k x_k + B_k u_k) - x_k^p \quad (20)$$

3. THEORY AND METHOD

3.1 Adaptive NMPC based on Linear Parameter Varying Model (LPV-MPC)

LPV-MPC adapts to changes in the system dynamics by adjusting the parameters of the model online. The model used is as follows:

$$x_{k+1} = A(\rho_k)x_k + B(\rho_k)x_k \quad (21)$$

$$\hat{y}_k = C(\rho_k)x_k \quad (22)$$

The system matrices in the LPV model are updated as:

$$A(\rho_k) = A_0 + \textstyle\sum_{i=1}^{N} A_i\, w_i(\rho_k) \quad (23)$$

$$B(\rho_k) = B_0 + \textstyle\sum_{i=1}^{N} B_i\, w_i(\rho_k) \quad (24)$$

$$C(\rho_k) = C_0 + \textstyle\sum_{i=1}^{N} C_i\, w_i(\rho_k) \quad (25)$$

where $w(\rho_k)$ is a validity function chosen as the Gaussian function:

$$w(\rho_k) = \exp\left[-\frac{1}{2}\frac{(\rho_k - \bar{X})^{\mathrm{T}}(\rho_k - \bar{X})}{\sigma^2}\right] \quad (26)$$

where ρ_k is a scheduling variable vector which characterizes the operating condition of the process. Typically, any of the process input or output is often recommended as the scheduling variable. $\bar{X}$ is the centre of the model and σ stands for the width of the validity function. The parameters in the validity function that is, $\bar{X}$ and σ are chosen by the designer.
To obtain the parameters of the LPV model using the global identification approach, the following optimization problem is solved, which consists of finding the model parameters, $\pi = \{A_i, B_i, C_i, D_i, \sigma_i\}$, which minimize the cost function:

$$\min_\pi \mathcal{J}_{sse}(k) = \sum_{k=1}^{N_T} \left(y_k^p - \hat{y}_k\right)^2 \quad (27)$$

where y_k^p and $\hat{y}_k$ stand for plant's output and LPV model's prediction, respectively, N_T is the number of data points used. To generate data for process identification, a multilevel random input is employed in exciting the process. The starting point for the optimization posed in Eq. (27) needs to be chosen correctly for fast convergence. In this study, the starting point for the optimization is proposed to be obtained as follows. The state space model parameters $\{A_i, B_i, C_i, D_i\}$ are selected as the linearized model of the nonlinear system at the nominal operating point while the Gaussian function parameter, σ is proposed to be chosen as a value which provides a measure of the variance of the scheduling variable.

3.2 NMPC based on Successive Linearization

The adaptive NMPC based on successive linearization (SL-MPC) relies on the linear time-varying (LTV) state space model:

$$x_{k+1} = f(x_k, u_{k-1}) + A_k\delta x_k + B_k\delta u_k \quad (28)$$

$$y_k = g(x_k) + C_k\delta x_k \quad (29)$$

where $A_k = \left.\frac{\partial f}{\partial x_k}\right|_{x_k,u_{k-1}}$, $B_k = \left.\frac{\partial f}{\partial x_k}\right|_{x_k,u_{k-1}}$, $C_k = \left.\frac{\partial g}{\partial x_k}\right|_{x_k,u_{k-1}}$

The state space matrices A_k, B_k, and C_k are updated online, quite unlike in the linear MPC where those matrices are constant. Figure 1 and algorithm 1 are for implementing LPV-MPC and SL-MPC methods.

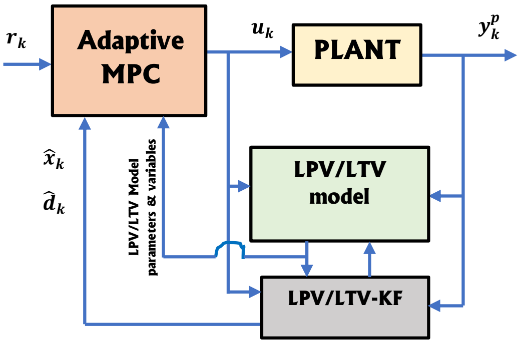

Fig. 1. Block diagram used to implement Adaptive NMPC

Algorithm 1: The algorithm for implementing adaptive NMPC (LPV-MPC or SL-MPC).

(1) Initialization: Initialize process variables.

(2) Main Loop: At every time step k:

Measurement: Obtain the input, u_k and the output, y_k^p of the process through measurement.

Weighting function computation for LPV-MPC: Compute the weighting function, $w(\rho_k)$ based on the measured scheduling variable, $\rho_k = u_k$ or y_k. Skip this step for SL-MPC.

LPV/LTV model parameter calculation: Compute the parameter-dependent state space matrices of the LPV model: $A(\rho_k)$, $B(\rho_k)$, $C(\rho_k)$ or LTV model: A_k, B_k, C_k accordingly.

Update Nominal variables: Update the values of the variables, x_{ss}, y_{ss}, u_{ss} and Δx_{ss}.

State and disturbance estimations: Estimate the system state, $\hat{x}_k$ and the disturbance state, $\hat{d}_k$ from the measured output and the computed LPV/LTV model parameters using the LPV/LTV model-based Kalman Filter (LPV/LTV-KF).

Control action: Solve the quadratic programming problem in Eqs. (8) to (19) and apply the first control move to the process.

Time step increment: Increment the sampling time from k to k+1 and repeat step 2 for the subsequent sampling time.

(3) Termination

4. SIMULATION RESULTS

Three simulation examples are employed to demonstrate the efficacy of the proposed approach. The assessment metrics include the integral of absolute error (IAE), the computational time required by the central processing unit for control law calculation (CPU-Time), and the total variation of the control signal (TVC), which was utilized to evaluate the control signal's quality.

4.1 Single Input Single Output Continuously Stirred Tank Reactor (SISO CSTR)

Consider a SISO CSTR (Bamimore, 2023) modelled in terms of dimensionless variables as follows:

$$\dot{x}_1 = -x_1 + D_a(1-x_1)\exp\left(\frac{x_2}{1+\frac{x_2}{\gamma}}\right) \quad (30)$$

$$\dot{x}_2 = -x_2 + BD_a(1-x_1)\exp\left(\frac{x_2}{1+\frac{x_2}{\gamma}}\right) + \vartheta(u - x_2) \quad (31)$$

$$y = x_2 \quad (32)$$

where x_1, x_2 and u stand for the dimensionless reagent conversion, the dimensionless reactor temperature (output), and the dimensionless coolant temperature, respectively. Their nominal values are: $B = 8.0$, $\beta = 0.3$, $x_{1f} = 1.0$, $q = 1.0$, $D_a = 0.072$, $\gamma = 20.0$, $x_{2f} = 0.0$, $x_3 = 0.0$. The control objective is to regulate the dimensionless concentration, x_1 using the dimensionless coolant temperature, u.

4.1.1 LPV model development for the SISO-CSTR

In order to identify LPV model for the SISO-CSTR using the proposed method, the nonlinear SIMULINK model of the CSTR was excited using a well-designed input. 2500 input-output-scheduling variable $(u - y - \rho)$ dataset was collected by sampling at 0.1 second, with the scheduling variable selected as $\rho_k = y_k$. By using the nominal linear model of the SISO-CSTR as the starting point for the optimization, the LPV model was identified using the "fminsearch" optimization routine in MATLAB:

$$A(\rho_k) = \begin{pmatrix} 0.912 & 0.00565 \\ -0.0569 & 0.974 \end{pmatrix} + w(\rho_k)\begin{pmatrix} 0.08522 & 0.000589 \\ -0.004181 & 0.1065 \end{pmatrix} \quad (33)$$

$$B(\rho_k) = \begin{pmatrix} 0.00008178 \\ 0.0311 \end{pmatrix} + w(\rho_k)\begin{pmatrix} 0.00000987 \\ 0.0031017 \end{pmatrix} \quad (34)$$

$$C(\rho_k) = \begin{pmatrix} 0 & 1 \end{pmatrix} \quad (35)$$

The scheduling variable is normalized using the weighting function

$$w(\rho_k) = \exp\left[-\frac{1}{2}\left(\frac{y_k - 3.5}{0.733}\right)^2\right] \quad (36)$$

After validation, the LPV model demonstrates a strong fit with a mean square error (MSE) value of 0.0037 compared with the linear model which has a MSE value of 0.096.

4.1.3 Adaptive NMPC Design for the SISO-CSTR

LPV-MPC was designed using the identified LPV model and the proposed algorithm 1 while SL-MPC was designed using online successive linearized (i.e., LTV) models. For comparison purpose, LMPC and NMPC were equally designed using the linearized model at the process nominal operating condition ($x_1 = 0.2159$, $x_2 = 1.4375, u = 0.473$) and the nonlinear model, respectively. The controller tuning for all four types of MPC were selected to be identical to ensure a fair comparison, i.e.: $N_p = 10$, $N_u = 2$, $W_y = 1$, $W_{\Delta u} = 0.5$ while the constraints were set as: $-10 \leq \Delta u_k \leq 10$, $-2 \leq u_k \leq 3$, and $0 \leq y_k \leq 6$. The servo performance of designed controllers was investigated in the face of process/model mismatch with the uncertain model parameter, β taken as 0.2 in the model and 0.3 in the plant. The simulation results comparing the four types of predictive controllers are plotted in Fig. 2 while the performance metrics are presented on Table 1. Both the figure and the table indicate that all the techniques track the varying setpoint very well, with the proposed LPV-MPC and SL-MPC exhibiting performances which closely match that of the NMPC, with an IAE of 40.1 and 38.2 respectively.

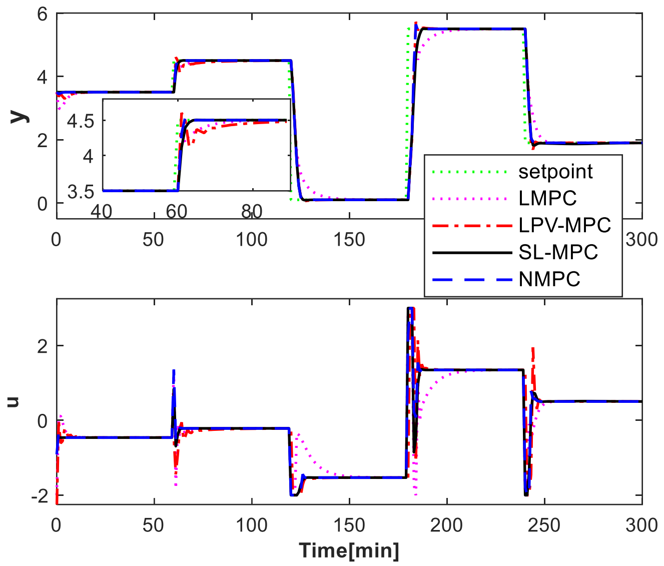

Fig. 2: Closed-loop response of the SISO-CSTR under process/model mismatch, with model $\beta = 0.2$, plant $\beta = 0.3$.

NMPC as expected has the best performance with an IAE of 34.8 while LMPC has the least performance with an IAE of 55.0. In terms of TVC, SL-MPC and NMPC have the smoothest control moves with TVC values of 22.5 and 22.8, respectively (which are almost the same), followed by LMPC with TVC value of 28.3 while LPV-MPC has a value of 32.3. In terms of computational efficiency, LMPC, SL-MPC and LPV-MPC have almost the same CPU time of ~0.01 sec while NMPC has the largest CPU time of 0.129 second. This implies that the proposed LPV-MPC and SL-MPC can be executed in real-time scenarios. The performance of the controllers was equally assessed under noisy measurements (with a variance of 0.005) and the results are summarized on Table 1. The same pattern of results is observed as discussed above.

4.2 Two Inputs Two Outputs Continuously stirred tank reactor (TITO CSTR)

Consider a non-isothermal Continuous Stirred Tank Reactor (CSTR) operating under constant volume conditions, with two inputs and two outputs (TITO CSTR). This reactor involves an irreversible first-order exothermic reaction, $A \rightarrow B$, and employs a single coolant stream for cooling purposes. The dynamics of this process are described by the following set of nonlinear ordinary differential equations (Bamimore, 2023):

$$\dot{C}_A(t) = \frac{q}{V}[C_{AO} - C_A(t)] - k_O C_A(t) exp\left(\frac{-E}{RT(t)}\right) \quad (37)$$

$$\dot{T}(t) = \frac{q}{V}\left(T_O - T(t)\right) - k_1 C_A(t) exp\left(\frac{-E}{RT(t)}\right)$$
$$+k_2 q_c(t)\left[1 - exp\left(\frac{-k_3}{q_c(t)}\right)\right][T_{c0} - T(t)] \quad (38)$$

where the reaction rate constants are

$$k_1 = \frac{(-\Delta H)k_O}{\rho c_p},\ k_2 = \frac{\rho_c c_{pc}}{\rho c_p V},\ k_3 = \frac{-hA}{\rho c_p} \quad (39)$$

More details about this process can be found in Bamimore (2023). The control objective is to regulate the measured concentration, C_A and temperature, T at their setpoints using the inlet flowrate, q and coolant flowrate q_c, respectively.

4.2.1 LPV model development for the TITO CSTR

In order to identify LPV model for the TITO CSTR, 500 input-output-scheduling variables $(u - y - \rho)$ dataset was collected by sampling at 0.1 second. The scheduling variables are selected as $\rho_k = [q_k \quad q_{ck}]^T$. By using the linearized model of the TITO CSTR as the starting point for the optimization, the LPV model was identified with the "fminsearch" optimization routine in MATLAB. After several iterations, the optimal model was obtained as follows:

$$A(\rho_k) = \begin{pmatrix} -0.000246 & -0.0000407 \\ 1.600 & 0.0155 \end{pmatrix} \times 10^2 + w(\rho_k)\begin{pmatrix} 0.000792 & -0.0002370 \\ -0.003385 & -0.002715 \end{pmatrix} \quad (40)$$

$$B(\rho_k) = \begin{pmatrix} 0.0003964 & 0.0004813 \\ -0.01766 & -0.1409 \end{pmatrix} + w(\rho_k)\begin{pmatrix} 0.27023 & -0.1841 \\ 0.2371 & 0.8580 \end{pmatrix} \times 10^{-3} \quad (41)$$

$$C(\rho_k) = diag(1,1) \quad (42)$$

The scheduling variable is normalized using the weighting function

$$w(\rho_k) = \exp\left[-\frac{1}{2}\left(\frac{q_k - 95}{40}\right)^2 - \frac{1}{2}\left(\frac{q_{ck} - 102.5}{15}\right)^2\right] \quad (43)$$

After validation, the LPV model demonstrates a strong fit with a MSE value of 0.2245 compared with the linear model which has a MSE value of 0.2730.

4.2.3 Adaptive NMPC Design for the TITO CSTR

Four different types of predictive controllers were designed with the tuning parameters and constraints selected as: $N_p = 20$, $N_u = 2$, $W_y = \begin{bmatrix} 4000 & 0 \\ 0 & 1 \end{bmatrix}$, $W_{\Delta u} = \begin{bmatrix} 1.5 & 0 \\ 0 & 1.5 \end{bmatrix}$, $\begin{bmatrix} -50 \\ -50 \end{bmatrix} \leq \begin{bmatrix} \Delta q_k \\ \Delta q_{c_k} \end{bmatrix} \leq \begin{bmatrix} 50 \\ 50 \end{bmatrix}$, $\begin{bmatrix} 80 \\ 60 \end{bmatrix} \leq \begin{bmatrix} q_k \\ q_{c_k} \end{bmatrix} \leq \begin{bmatrix} 160 \\ 120 \end{bmatrix}$, $\begin{bmatrix} 0.02 \\ 430 \end{bmatrix} \leq \begin{bmatrix} C_{A_k} \\ T_k \end{bmatrix} \leq \begin{bmatrix} 0.15 \\ 490 \end{bmatrix}$. The servo performance of the controllers was investigated in the presence of process/model mismatch, with model k_o=75% of nominal value, plant k_o=nominal value. The simulation results comparing the four methods are plotted in Figure 3 with the performance metrics summarized in Table 1. The results reveal that LPV-MPC demonstrate the best servo performances with an IAE of 43.5, followed by SL-MPC and NMPC in this order with respective IAE of 58.8 and 60.7. LPMC has the worst performance. The average CPU time are summarized in Table 1. The results reveal that LMPC is the most computationally efficient with a CPU time of 0.028 sec, followed by the two adaptive methods with respective CPU times of 0.093 and 0.127 for LPV-MPC and SL-MPC. However, NMPC is the most computationally intensive with a CPU time of 1.7 second. The performance of the controllers was equally investigated under noisy measurements (with variances of 1×10^{-6} and 0.75 for C_A and T, respectively) and the results summarized on Table 1. The results followed the same pattern as discussed above.

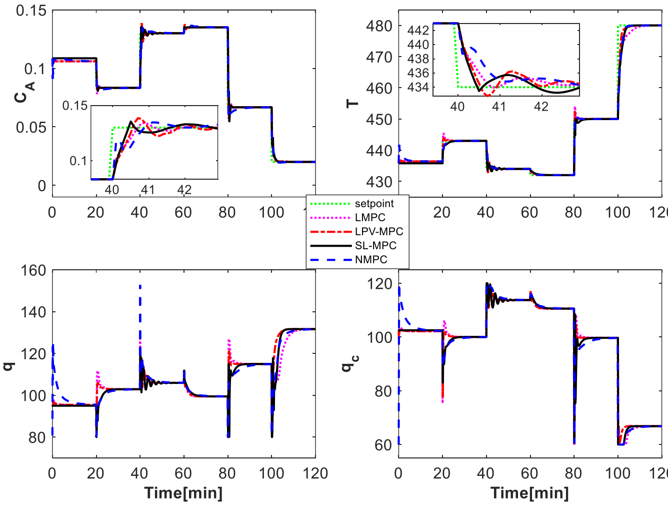

Fig. 3. Closed-loop response of the TITO-CSTR under process/model mismatch, with model k_o=75% of nominal value, plant k_o=nominal value.

4.3 Forced Evaporator

The forced evaporator process (Maciejowski, 2002) consists of a feed stream entering the evaporator (Figure 4) at flow rate F_1, concentration X_1 and temperature T_1. It is mixed with recirculating liquor, pumped through the evaporator at a flow rate F_3. The evaporator itself, which is a heat exchanger, is heated by steam flowing at the rate of F_{100}. The separator Level (L_2) is determined by equation (44)

$$\rho A \frac{dL_2}{dt} = F_1 - F_2 - F_4 \quad (44)$$

where ρ is the liquid density and A is the cross-sectional area of the separator. The evaporator itself is modelled by five equations (45) – (49):

$$M \frac{dX_2}{dt} = F_1 \times X_1 - F_2 \times X_2 \quad (45)$$

$$C \frac{dP_2}{dt} = F_4 - F_5 \quad (46)$$

$$T_2 = 0.5616 + 0.3126 X_2 + 48.43 \quad (47)$$

$$T_3 = 0.507 P_2 + 55.0 \quad (48)$$

$$F_4 = \frac{Q100 - F_1 \times C_p (T_2 - T_1)}{\lambda} \quad (49)$$

where X_1 and X_2 are feed and product composition.

The heater steam jacket is described by equations (50) – (52)

$$T100 = 0.1538 \times P100 + 90.0 \quad (50)$$

$$Q100 = 0.16(F_1 + F_3)(T100 - T_2) \quad (51)$$

$$F100 = \frac{Q100}{\lambda_s} \quad (52)$$

The condenser is described by equations (53) – (55)

$$Q200 = \frac{UA_2(T_3 - T200)}{1 + \frac{UA_2}{2 C_p F200}} \quad (53)$$

$$T201 = T200 + \frac{Q200}{F200 \times C_p} \quad (54)$$

$$F_5 = \frac{Q200}{\lambda} \quad (55)$$

The details about the modelling, the process variables and parameters can be found in Maciejowski (2002). The controlled objective is to maintain liquid level, L_2 in the evaporator at a constant value of 1m while tracking ramp setpoint changes in X_2 from 25% to 15% and P_2 from 50.5kPa to 70kPa using F_2, P_{100} and F_{200}, respectively as the manipulated variables.

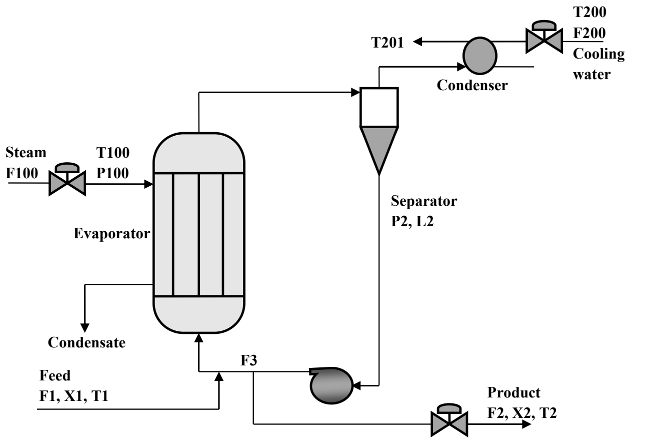

Fig. 4. Forced-circulation evaporator

4.3.1 *LPV model development for the forced evaporator*

The scheduling variables for this process were chosen as $\rho_k = [F_{2_k} \quad P_{100_k} \quad F_{200_k}]^T$. In order to identify an LPV model for the process, 1500 $(u - y - \rho)$ dataset was collected, sampled at 1 second. Using the "fminsearch" algorithm in MATLAB and with the state-space parameters obtained at nominal operating point as the starting point, the final LPV model is:

$$A(\rho_k) = \begin{pmatrix} 1.014 & 0.0042 & 0.0091 \\ -0.0024 & 0.8876 & -0.0007 \\ -0.0004 & -0.0240 & 0.9368 \end{pmatrix} + w(\rho_k) \begin{pmatrix} 0.0146 & 0 & 0.0001 \\ -0.0001 & 0.0088 & 0.0007 \\ 0.0008 & -0.0002 & 0.0100 \end{pmatrix} \quad (56)$$

$$B(\rho_k) = \begin{pmatrix} -0.0538 & -0.0020 & 0 \\ -0.9969 & 0 & -0.0016 \\ 0.0107 & 0.0105 & -0.0031 \end{pmatrix} + w(\rho_k) \begin{pmatrix} -0.0004 & 0 & 0 \\ 0.0095 & 0.0006 & 0.0006 \\ 0.0001 & 0.0001 & -0.0001 \end{pmatrix} \quad (57)$$

$$C(\rho_k) = diag(1,1,1) \quad (58)$$

The weighting function is given as:

$$w(\rho_k) = \exp\left[\left(\frac{-0.5 \times (F_{2_k} - 2.00)}{0.1729}\right)^2 + \left(\frac{-0.5 \times (P_{100_k} - 193)}{0.1414}\right)^2 + \left(\frac{-0.5 \times (F_{200_k} - 213)}{0.2672}\right)^2\right] \quad (59)$$

After validation, the LPV model demonstrates a strong fit with a MSE of 8.4 compared with the linear model which has an MSE of 18.5.

4.3.2 *Adaptive NMPC Design for the forced evaporator*

Four types of predictive controllers were designed for the forced evaporator process with the tuning parameters selected as: $N_p = 30$, $N_u = 3$, $W_y = \text{diag}(5000,10,10)$, $W_{\Delta u} = \text{diag}(1,0.01,0.01)$. The process constraints were selected as found in (Maciejowski, 2002). Predictive controllers were implemented on the nonlinear Simulink model of the evaporator process. The simulation results are plotted in Figure 5, while the computed performance metrics are summarized on Table 1. The results indicate that the NMPC exhibits superior servo performance overall with an IAE of 120.5 and TVC of 632.3, followed by SL-MPC with an IAE of 163.9 and TVC of 572.1 and then followed by LPV-MPC with an IAE of 201.8 and TVC of 765.8. LMPC has the least performance with an IAE of 216.6 and TVC of 698.5. However, SL-MPC has the smoothest control signal as measured by a TVC of 572.1. The two adaptive methods (LPV-MPC and SL-MPC) are computationally efficient with CPU time of 0.027 sec and 0.024 sec respectively. NMPC is the most computationally demanding with CPU time of 0.442 seconds.

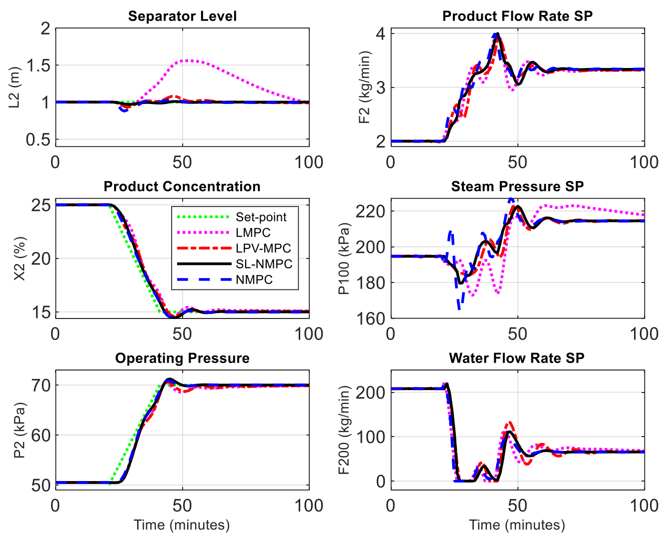

Fig. 5. Closed-loop responses of the forced evaporator.

The controllers' performance was equally evaluated under conditions of noisy measurement (with variances of 1 for disturbance variables: F_1, X_1, T_1 and T_{200}) and the results summarized on Table 1. The same pattern of results is observed as discussed above.

Table 1: Performance Indices

	IAE	TVC	CPU(s)	IAE	TVC	CPU(s)
	SISO-CSTR			**SISO-CSTR with noises**		
LMPC	55.0	36.3	0.012	61.3	483.7	0.014
LPV-MPC	40.1	32.3	0.011	57.6	142.1	0.012
SL-MPC	38.2	22.5	0.010	51.1	87.3	0.028
NMPC	34.8	22.8	0.129	50.6	589.3	0.293
	TITO-CSTR			**TITO-CSTR with noises**		
LMPC	62.3	490	0.028	134	1370	0.026
LPV-MPC	40.8	473	0.093	119	1375	0.182
SL-MPC	58.8	737	0.127	132	2487	0.214
NMPC	60.7	689	1.70	133	1592	1.604
	Evaporator			**Evaporator with noises**		
LMPC	216.6	698.5	0.018	147	654	0.020
LPV-MPC	201.8	765.8	0.027	133	682	0.040
SL-MPC	163.9	572.1	0.024	85	505	0.037
NMPC	120.5	632.3	0.442	40	760	0.507

5.0 CONCLUSIONS

This study presents the results of a comparative study between two adaptive NMPC approaches: LPV-MPC and SL-MPC. Three simulation examples were used to demonstrate the comparison. The identified LPV model found by optimization with linear model as the starting point gave a very good fit. The simulation results obtained showed that the two adaptive predictive control methods demonstrate similar servo performance as measured by their IAE and TVC values. The two methods were also found to be computationally less demanding than traditional NMPC as measured by their CPU time. A possible future research direction may be to find a more effective way of identifying LPV model especially for large scale systems.

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